MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EME4076 – MECHANICAL VIBRATIONS

(ME)

1 MARCH 2019
3:00 p.m. – 5:00 p.m.
2 Hours

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows an overhanging steel beam ABC which is used to support a block of mass $m = 225$ kg at the end of the beam. The beam is pinned at A and B, and has a flexural modulus of $EI = 2.6\times10^5$ N-m^2, where E and I are respectively Young's modulus and moment of inertia of the cross section of the beam. The pin A of the beam is subjected to a vertical motion of $y_A = Y\sin\omega t$, where $Y = 2.5$ mm. Neglect the mass of the beam.

(a) If the block does not leave the beam, determine the range of excitation frequencies ω for which the amplitude of motion of the block will be less than 5 mm.

[12 marks]

(b) Determine the excitation frequencies ω at which the block is about to loss contact to the beam. Find the time when it happens?
Hint: to lose contact, the reaction force from the beam to the block must be zero.

[13 marks]

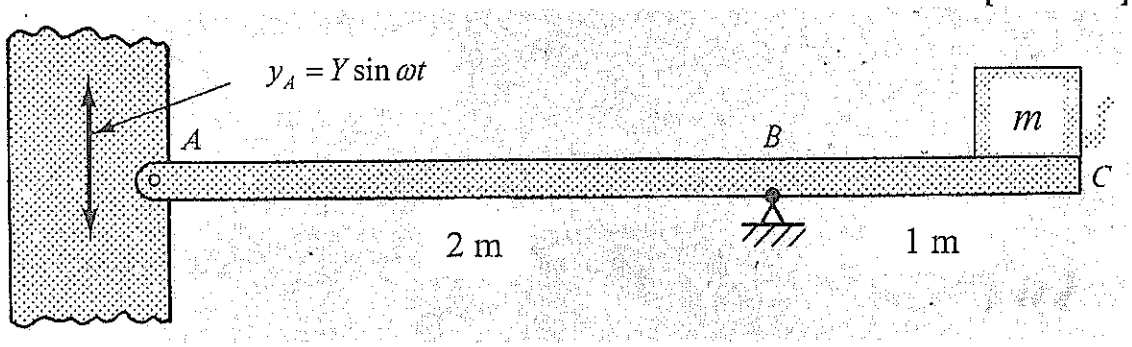


Figure Q1

Continued .....

Question 2

(a) Figure Q2(a) shows a vehicle which is used to transport an instrumentation package. The package is tied to the floor of the vehicle. Accelerometer measurement indicates that when the vehicle is traveling on a 36 rad/s frequency sinusoidal road at its normal speed of 85 m/s, the dominant component of the vehicle's vertical acceleration is found to be $8.5\,\text{m/s}^2$. Design an isolator system with a damping ratio of 0.05 such that the acceleration amplitude of the 60-kg instrument is at most $0.5\,\text{m/s}^2$. With this isolator, what is the maximum displacement of the instrument? **Hint:** $T_d = \omega^2 X/\omega^2 Y$.

[13 marks]

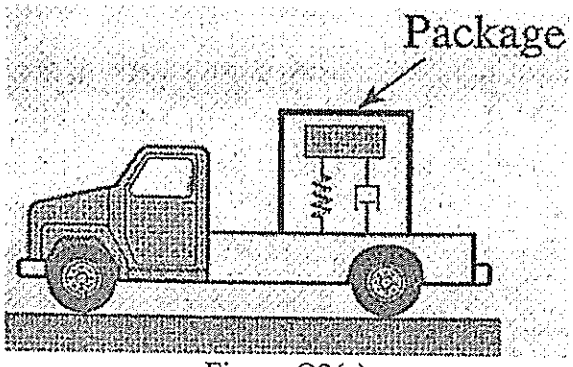


Figure Q2(a)

(b) A number of undamped isolators are going to be used in between a 110-kg machine and its foundation. Each of the isolators has identical stiffness of 6×10^5 N/m. Figure Q2(b) shows the harmonic excitation of the machine. If the isolators are placed in series, determine the largest number of isolators that are required such that the maximum force that is allowed to transmit to the foundation is limited to 5000 N. If these number of isolators are arranged in parallel, determine the percentage of isolation provided by them and comment your answer.

[12 marks]

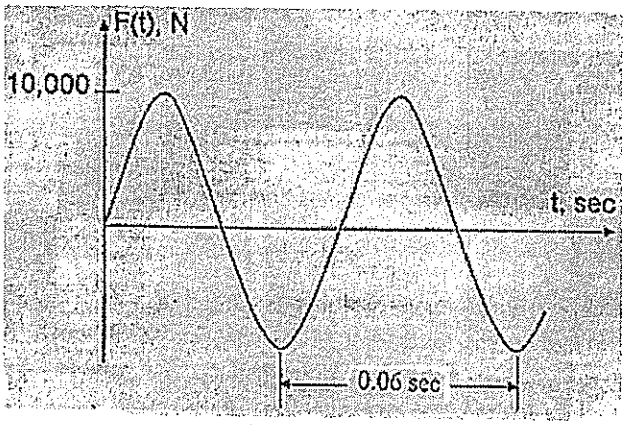


Figure Q2(b)

Continued .....

Question 3

Figure Q3 shows a hot air balloon of mass m which is used to lift a cargo of passengers of weight Mg in a basket through 12 equally spaced elastic ropes. Each of the ropes has a stiffness of k and making 45^0 with vertical axis of the balloon. Turning on or off of the burner of the balloon will cause the balloon to go upward or come down slowly. This will cause the balloon to be subjected a sinusoidal load with amplitude A and frequency ω_f.

(a) Derive the equations of motion of the system in terms of x and y, where x and y are vertical displacements of the balloon and the cargo, respectively.

[10 marks]

(b) Determine the frequency of turning on and off the burner, ω_f, that will cause the system to be in a critical state. that is in resonance.

[7 marks]

(c) Find the maximum tension in each rope.

[8 marks]

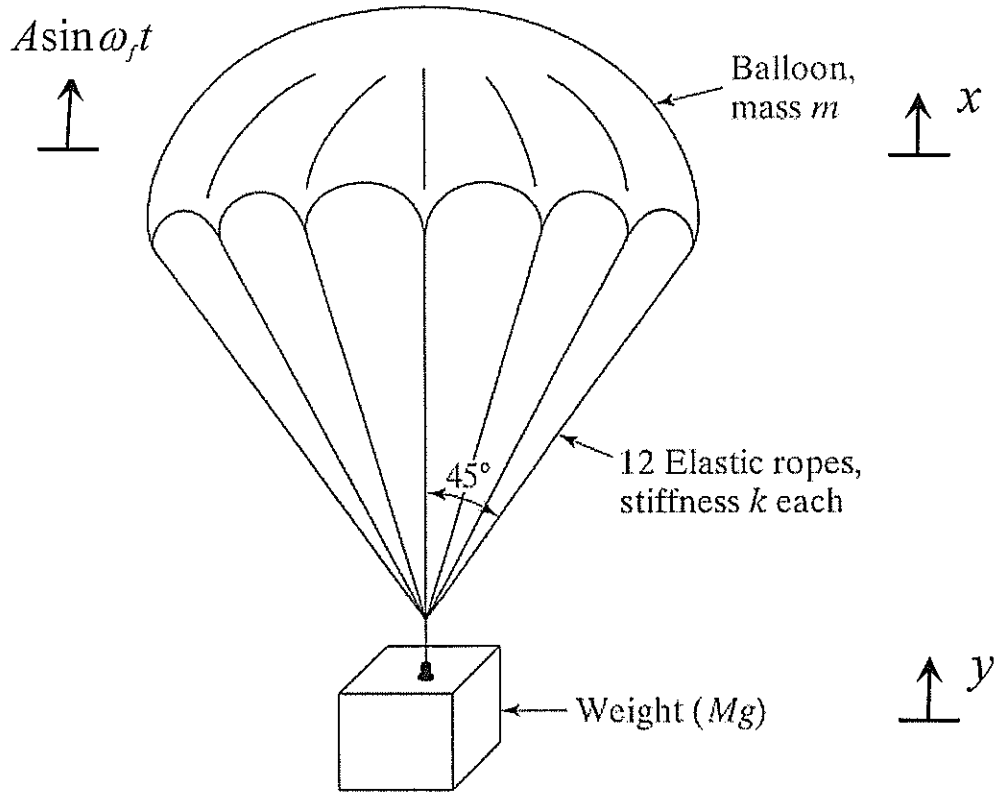


Figure Q3

Continued .....

Question 4

Figure Q4 shows a new model automobile which is suspended using inextensible cables attached to the front and rear axles. The system is in equilibrium at the instant shown in the figure. The automobile has a mass m = 1300 kg and its center of gravity G is located at a distance h above a line connecting the front and rear axles.

(a) Derive the dynamic equation of motion of the system for small oscillation.

[7 marks]

(b) From testing, it is found out that when $L = 4$ m, the period of oscillation of the system is 4.04 s. However, if L is decreased to 3 m, the period is shortened to 3.54 s. Determine the h and radius of gyration of the system about G, r_G.

Hint: $mr_G^2 = I_G$, I_G = mass moment of inertia of the automobile about G.

[12 marks]

(c) Find the maximum natural frequency of the system.

[6 marks]

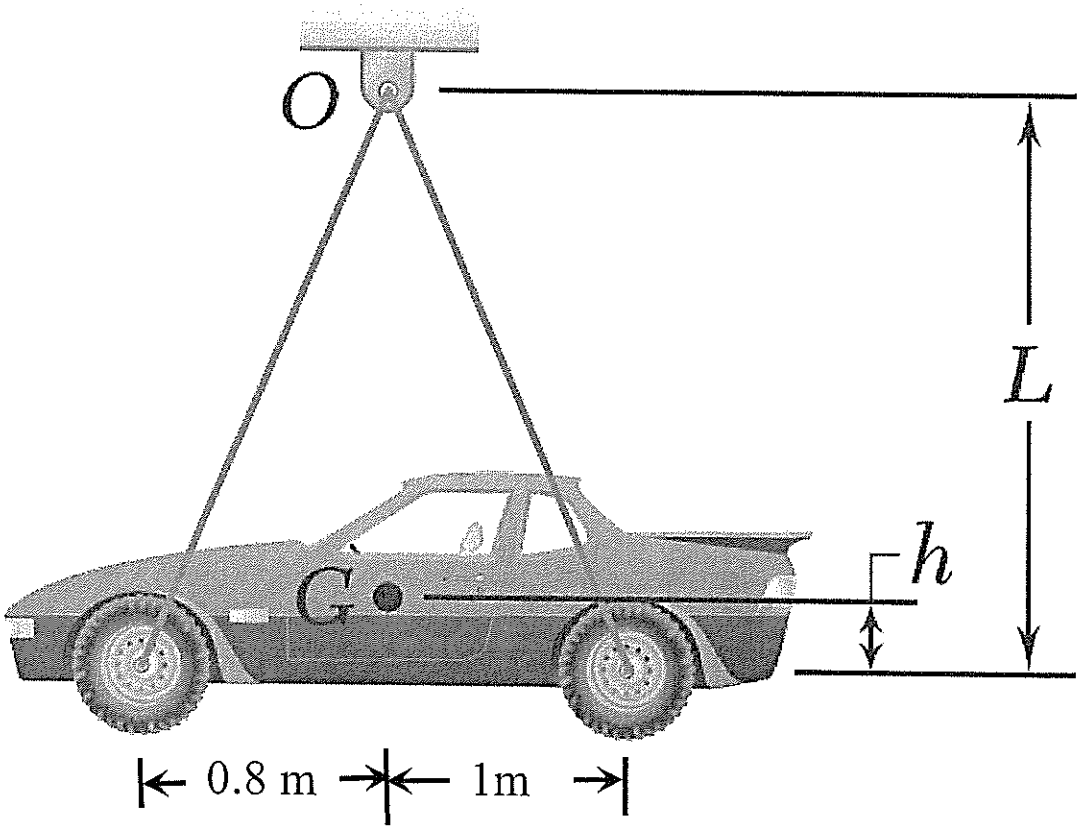


Figure Q4

Continued .....

Question 5

Figure Q5(i) shows a quarter car model of an automobile which is moving at a constant speed of v_0 towards a toll booth. Figure Q5(ii) shows the wake-up strips with a height of Y_0 before the toll booth, which are used to caution the driver of the car to drive carefully as he/she approaches the booth.

(a) Determine the width of the first strip such that the response of the car in between the first and second strip is maximized.
Hint: use identity $\sin(a)-\sin(b)=2\cos\left((a+b)/2\right)\sin\left((a-b)/2\right)$.

[12 marks]

(b) Locate the second strip such that the response of the car after the time t_1+t_0 is minimized.

[13 marks]

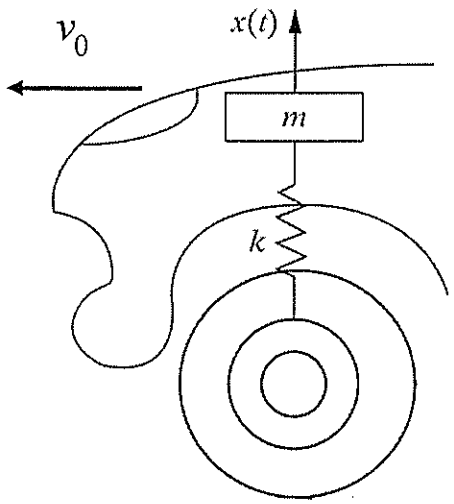


Figure Q5(i)

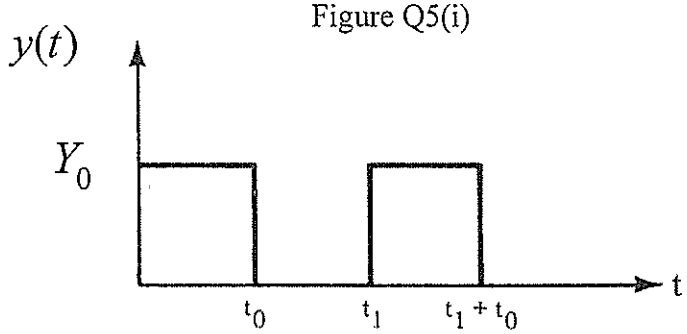


Figure Q5(ii)

Continued .....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D\sin(\omega t - \varphi)$$

$$D = \frac{F_0/k}{\sqrt{[1-r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1}\frac{2\zeta r}{1-r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct)e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0}\frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n\sin n\omega t]$$

$$a_n = \frac{2}{\tau}\int_0^{\tau} x(t)\cos n\omega t\, dt \qquad b_n = \frac{2}{\tau}\int_0^{\tau} x(t)\sin n\omega t\, dt$$

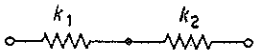
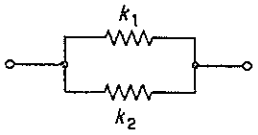
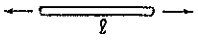
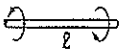
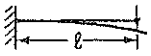
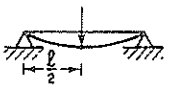
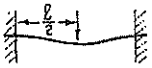
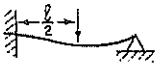
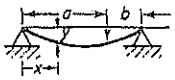
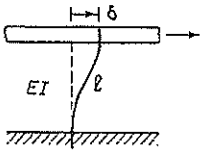
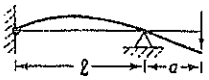
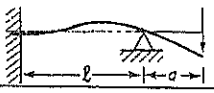
Unit response function (underdamped)

$$h(t-\tau) = \frac{e^{-\xi\omega_n(t-\tau)}}{m\omega_d}\sin\omega_d(t-\tau)$$

Continued .....

Appendix 2

Table of Spring Stiffness

	$k = \dfrac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \dfrac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \dfrac{EA}{l}$	A = cross-sectional area
	$k = \dfrac{GJ}{l}$	J = torsion constant of cross section
	$k = \dfrac{Gd^4}{64nR^3}$	n = number of turns
	$k = \dfrac{3EI}{l^3}$	k at position of load
	$k = \dfrac{48EI}{l^3}$	
	$k = \dfrac{192EI}{l^3}$	
	$k = \dfrac{768EI}{7l^3}$	
	$k = \dfrac{3EIl}{a^2b^2}$	$y_x = \dfrac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \dfrac{12EI}{l^3}$	
	$k = \dfrac{3EI}{(l + a)a^2}$	
	$k = \dfrac{24EI}{a^2(3l + 8a)}$	

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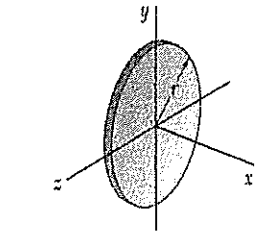
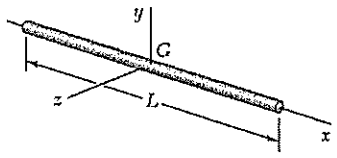
Appendix 3: Mass Moment of Inertia of Homogenous Solids

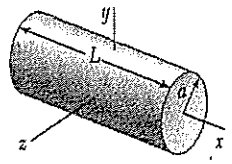


$I_x = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$

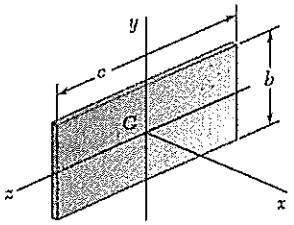


$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{2}ma^2$

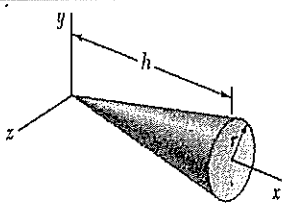
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



$I_x = \frac{1}{12}m(b^2 + c^2)$

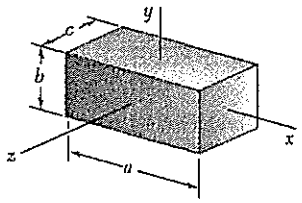
$I_y = \frac{1}{12}mc^2$

$I_z = \frac{1}{12}mb^2$



$I_x = \frac{3}{10}ma^2$

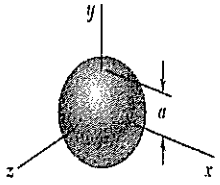
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}m(c^2 + a^2)$

$I_z = \frac{1}{12}m(a^2 + b^2)$



$I_x = I_y = I_z = \frac{2}{5}ma^2$

End of Paper